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Find the radius of convergence of the series

Mathematical concept In mathematics, the radius of convergence of a power series is the radius of the largest disk in which the series converges. It is either a non-negative real number or ∞ {\displaystyle \infty } . When it is positive, the power series converges absolutely and uniformly on compact sets inside the open disk of radius equal to the radius of convergence, and it is the Taylor series of the analytic function to which it converges. Definition For a power series ƒ defined as: f (z) = ∑ n = 0 ∞ c n (z − a) n , {\displaystyle f(z)=\sum _{n=0}^{\infty }c_{n}(z-a)^{n},} where, a is a complex constant, the center of the disk of convergence, cn is the nth complex coefficient, and z is a complex variable. The radius of convergence r is a nonnegative real number or ∞ {\displaystyle \infty } such that the series converges if | z − a | < r {\displaystyle |z-a| r . {\displaystyle |z-a|>r.\,} Some may prefer an alternative definition, as existence is obvious: r = sup { | z − a | | ∑ n = 0 ∞ c n (z − a) n converges } {\displaystyle r=\sup \left\{|z-a|\ \left|\ \sum _{n=0}^{\infty }c_{n}(z-a)^{n}\ {\text{ converges }}\right.\right\}} On the boundary, that is, where |z − a| = r, the behavior of the power series may be complicated, and the series may converge for some values of z and diverge for others. The radius of convergence is infinite if the series converges for all complex numbers z.[1] Finding the radius of convergence Two cases arise. The first case is theoretical: when you know all the coefficients c n {\displaystyle c_{n}} then you take certain limits and find the precise radius of convergence. The second case is practical: when you construct a power series solution of a difficult problem you typically will only know a finite number of terms in a power series, anywhere from a couple of terms to a hundred terms. In this second case, extrapolating a plot estimates the radius of convergence. Theoretical radius The radius of convergence can be found by applying the root test to the terms of the series. The root test uses the number C = lim sup n → ∞ | c n (z − a) n | n = lim sup n → ∞ (| c n | n) | z − a | {\displaystyle C=\limsup _{n\rightarrow \infty }{\sqrt[{n}]{|c_{n}(z-a)^{n}|}}=\limsup _{n\rightarrow \infty }\left({\sqrt[{n}]{|c_{n}|}}\right)|z-a|} "lim sup" denotes the limit superior. The root test states that the series converges if C 1. It follows that the power series converges if the distance from z to the center a is less than r = 1 lim sup n → ∞ | c n | n {\displaystyle r={\frac {1}{\limsup _{n\rightarrow \infty }{\sqrt[{n}]{|c_{n}|}}}}} and diverges if the distance exceeds that number; this statement is the Cauchy–Hadamard theorem. Note that r = 1/0 is interpreted as an infinite radius, meaning that ƒ is an entire function. The limit involved in the ratio test is usually easier to compute, and when that limit exists, it shows that the radius of convergence is finite. r = lim n → ∞ | c n c n + 1 | . {\displaystyle r=\lim _{n\rightarrow \infty }\left|{\frac {c_{n}}{c_{n+1}}}\right|.} This is shown as follows. The ratio test says the series converges if lim n → ∞ | c n + 1 (z − a) n + 1 | | c n (z − a) n | < 1. {\displaystyle \lim _{n\to \infty }{\frac {|c_{n+1}(z-a)^{n+1}|}{|c_{n}(z-a)^{n}|}}